International Journal of Theoretical Physics, Vol. 37, No. 1, 1998

Asymptotic State Vector Collapse and QED Nonequivalent Representations

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Received July 4, 1997

The state vector evolution in the interaction of a measured pure state with a collective quantum system or a field is analyzed in a nonperturbative QED formalism. As an example, the measurement of the electron final state scattered on a nucleus or neutrino is considered. The produced electromagnetic bremsstrahlung contains an unrestricted number of soft photons resulting in the total radiation flux becoming a classical observable, which means the state vector collapse. The evolution from the initial to the final system state is nonunitary and formally irreversible in the limit of infinite time.

1. INTRODUCTION

The problem of the description of state vector collapse in quantum mechanics (QM) is still open despite the multitude of proposed models and hypotheses (D'Espagnat, 1990). This paper analyzes some microscopic dynamical models of the collapse, i.e., the models which attempt to describe the interaction and the joint evolution of the measured state (particle) and the measuring device D (detector) from first QM principles. Currently the most popular models are the different variants of decoherence models, which take into account also the interaction of the environment E with a very large number of degrees of freedom (NDF) and D with small NDF (Zurek, 1982). Yet this model meets the serious conceptual difficulties summed up in the so-called environment observables paradox (EOP) (D'Espagnat, 1990). For any decoherence process at any time instant at least one observable $\hat{B}$ exists whose expectation value coincides with the value for the pure state and differs largely from the predicted value for the collapsed mixed state. Moreover, it follows that in principle it is possible to restore the system's initial state,

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0020-7748/98/0100-040 1$15.00/0

which contradicts with the irreversibility expected for the collapse. In general the EOP can be regarded as an important criterion for the correctness of measurement models.

Meanwhile it has been proposed that due to the very large internal NDF of real macroscopic detectors, the problem can be resolved by the methods of nonperturbative quantum field theory (QFT), which studies the dynamics of systems with infinite NDF (Neeman, 1986; Fukuda, 1987). The state manifold of such systems is described by unitarily nonequivalent (UN) representations, which enable us to resolve the EOP, as will be demonstrated below.

The main difficulty of this approach is that it can be correctly applied only for measurements on systems with not simply very large, but exactly infinite NDF. Practical measuring devices must have a finite mass and energy. Here we consider a QED bremsstrahlung model of the collapse which satisfies all these demands simultaneously and without contradictions. It makes it evident also that the collapse-like processes can occur not only on macroscopic objects, but also on the fundamental level of elementary particles and fields.

Nonperturbative methods in QED have been applied successfully for the study of the photon bremsstrahlung produced in any process of charged particle scattering on some target. The total number of produced photons with energy larger than k_0 is proportional to $e^2 \ln(P_e/k_0)$ i.e., it grows unrestrictedly if $k_0 \to 0$. The perturbative Feynman diagram method by definition works only for processes which yield small probabilities, whereas in the present case they approximate to 1 (Itzykson and Zuber, 1980). The nonperturbative formalism was developed initially for the semiclassical case when the motion of the charges is prescribed (classical) and the backreaction of the radiated electromagnetic (em) field $\hat{A}_\mu(x)$ on the charge motion can be neglected—BRF condition (Friedrichs, 1953). Consequently, in this case the electromagnetic current $J_\mu(x)$ is not an operator, but is c-valued, and for single-electron scattering its 4-dimensional Fourier transform is given by

$$J_\mu(k) = ie\left(\frac{p_\mu}{pk} - \frac{p'_\mu}{p'k}\right) \tag{1}$$

where p, p' are the initial and final electron 4-momenta, respectively. In this case the BRF condition means that the sum of the momentum of the radiated photons $|\mathbf{k}_s|$ is much less than the electron momentum transfer in the scattering $|\mathbf{p} - \mathbf{p}'|$ (Akhiezer and Berestetsky, 1981).

The final em field state is found by the nonperturbative computation of the S-matrix (S-operator). The T-product of the interaction Hamiltonian density is $\hat{H}_i(x) = \hat{H}_{em}(x) = \hat{J}_\mu(x)\,\hat{A}_\mu(x)$. Here $\hat{A}_\mu(x)$ is taken in the Feynman gauge with indefinite metric. The commutator of $\hat{H}_i(x)$, $\hat{H}_i(x')$, is a c-valued function. For the considered c-value currents J_μ which permits us to transform the T-product into a product of the integrals over 4-space we obtain

$$\hat{S}_{\rm em}(J) = \exp[i\phi(J) - i \int \hat{H}_i(x)\, d^4x] = \exp[(i-1)V(J) + U(J)] \quad (2)$$

where

$$U(J) = i \sum_{\lambda=1,2} \int d\tilde{k}\, [J_\mu(\mathbf{k}) e^\lambda_\mu a^+(\lambda, \mathbf{k}) - J^*_\mu(\mathbf{k}) e^\lambda_\mu a(\lambda, \mathbf{k})] \quad (3)$$

and

$$V(J) = \frac{1}{2(2\pi)^3} \int d\tilde{k}\, J^*_\mu(\mathbf{k}) J_\mu(\mathbf{k})$$

with $d\tilde{k} = d^3k/k_0$ and $a(\lambda, \mathbf{k})$ is the photon annihilation operator (Friedrichs 1953). Below we will omit the sum over the photon polarization index λ or the polarization vectors $e^{\lambda\mu}$ when it is unimportant. $\phi(J) = V(J)$ is equal to the quantum phase between the incoming and the outgoing states if the relation $J^*{}_\mu(k) = J_\mu\,(-k)$ is fulfilled, which is true for J_μ of (1). It is easy to verify from (3) that the amplitudes of the production of photons with different momenta $\mathbf{k}$ are independent. If the initial em field state is the vacuum $|\gamma_0\rangle = |0\rangle$, then the average number of produced photons is $d\bar{N}_{\mathbf{k}} = c|J_\mu(\mathbf{k})|^2$ $d\tilde{k}$. The action of $\hat{S}_{em}\,(J)$ results in divergent photon spectra $d\bar{N}_\gamma = c\; dk_o/k_0$ for $J_\mu(k)$ of (1). This means that the final asymptotic state $|f\rangle$ includes an infinite number of very soft photons whose total energy is finite (Jauch and Rohrlich, 1954). At the same time it yields

$$|\langle f|0\rangle| = \exp[-V(J)] = \exp(-\bar{N}_\gamma/2)$$

In conclusion it follows that the state $|f\rangle$ does not belong to the initial photon Fock space $H_{\rm F}$, but to a different Hilbert space orthogonal to H_F. So the complete field state manifold M_c becomes nonseparable, i.e., it has to be described by the tensor product of infinitely many Hilbert spaces H_i, each of them having its own cyclic vector-vacuum state $|0\rangle_i$. Any state of M_c is defined by two indices $|\psi_j\rangle_i$; $i = 0$, corresponds to H_F. Recall that any Hermitian operator $\hat{B}$-observable transforms only vectors inside the same Hilbert space $|\psi_2\rangle_i = \hat{B}\,|\psi_1\rangle_i$, and due to this, for arbitrary $|\psi_1\rangle_i$, $|\psi_2\rangle_i$, $i \neq l$, ${}_i\langle\psi_1|\hat{B}\,|\psi_2\rangle_l = 0$. So, if the final state is a superposition of states from different spaces $|f\rangle = |f_1\rangle_i + |f_2\rangle_l$ the interference terms (IT) for any $\hat{B}$ between $|f_1\rangle_i$ and $|f_2\rangle_l$ are equal to zero. Consequently, any measurement performed on such disjoint states cannot distinguish between the mixed and the pure initial states, which permit us to resolve the mentioned EOP for the UN representations. Note that the bremsstrahlung due to the classical motion of the charge results in a final em field state which can belong only to a single Hilbert space H_i. Hence, to obtain the final disjoint states described by QED the formalism must be extended to incorporate the bremsstrahlung of the charged particle state superpositions, which will be done in this paper.

The transition from H_F to some H_i corresponds to the Bogolubov boson transformation of the free field operators $a(\lambda, \mathbf{k})$ and $a^+(\lambda, \mathbf{k})$,

$$b(\lambda, \mathbf{k}) = a(\lambda, \mathbf{k}) + iJ_\mu(\mathbf{k})e^\lambda_\mu \tag{4}$$

which is nonunitary for $J_\mu(\mathbf{k})$ of (1), but conserves the vector norm $\langle f|f\rangle = \langle 0|0\rangle$.

2. QED MEASUREMENT MODEL

As an example of the collapse induced by the bremsstrahlung, we will consider weak scattering of an electron on a neutral particle (e.g., a neutrino) ν with mass m_0; the latter in principle can be zero. We also could consider the Coulomb scattering of an electron on a nucleus, but the infinite range of the potential results in an infinite total cross section, which involves unnecessary complications. We will show that the final disjoint states resulting from photon bremsstrahlung formally indicate whether the act of scattering took place or the particles passed unscattered and kept their initial states. At the same time this measurement gives the electron helicity λ_e, because for its left or right helicities the cross sections fulfill $\sigma_L >> \sigma_R$ in weak interactions.

Now the electron motion is nonclassical and defined by electron field operators. The general S-operator for $\hat{H}_i(x) = \hat{H}_{em}(x) + \hat{H}_w(x)$ should be determined. Here we will describe the method to calculate the matrix elements $\langle f|\hat{S}|i\rangle$ for the states of interest without writing down the S-operator in analytical form, which would be quite difficult. These nonperturbative calculations are possible for the soft photon radiation for which the BRF condition is fulfilled, i.e., the total em field recoil is much less than the electon momentum transferred to ν, as discussed in Section 1. This means that $\hat{H}_{em}(x)$ does not act on the electron field operators, conserving spin and momentum, and acts only on em field operators (Jauch and Rohrlich, 1954). On the contrary, $\hat{H}_w$ acts only on the e, ν fields, and due to this it is possible to factorize the S-operator into $\hat{S}_w$ and $\hat{S}_{em}$ parts. $\hat{S}_w$ defines the skeleton diagram which defines solely the final e', ν' states, which are dressed by the soft radiation given by $\hat{S}_{em}$. In turn $\hat{S}_{em}$ and consequently the final radiation field depends on the final electron momentum defined by the $\hat{S}_w$ action on the initial state.

So we start from the calculation of the $\hat{S}_w$ action on the initial e, ν states, neglecting $\hat{H}_{em}(x)$. The smallness of the weak interaction constant G permits us to calculate $\hat{S}_w$ perturbatevely with a good accuracy, and at c.m.s. energies below 1 TeV, which we will consider here. Its calculation can be restricted to the first-order diagram (Cheng and Li, 1984). Its amplitude M_w for the weak vertex e, $\nu \to e'$, ν' results in the spherically symmetric distribution of e', ν' in the c.m.s:

$$M_w(e', \nu') = \langle e, \nu|\hat{S}^1_w|e', \nu'\rangle = \frac{G}{\sqrt{2}} J_{L\mu}J^*_{L\mu} = \overline{u}'_e\gamma_\mu(1 + \gamma_5)u_e\overline{u}'_\nu\gamma_\mu(1 + \gamma_5)u_\nu \tag{5}$$

From this we can find the final em field state if we know the operator $\hat{S}_{em}$ (J^l) for the initial and final momentum eigenstates $|e\rangle$, $|e'_l\rangle$. Despite the fact that now the electron electromagnetic current is formally the operator, it was found that $\hat{S}_{em}$ (J^l) is independent of the initial and final electron polarizations and described by (2), in which current Fourier transform is equal to $J'^{l}_{\mu} = J_\mu$ (k, p, p'_l) of (1), where p, p'_l are the corresponding eigenvalues (Jauch and Rohrlich, 1954). This result does not seem surprising, because such states describe the prescribed electron motion in the phase space. Then, as follows from the superposition principle, if the final momentum eigenstate of the electron $|e'_l\rangle$ has the amplitude c_l, the final system state is

$$|\psi_f\rangle = \sum c_l|e'_l\rangle|\nu'_l\rangle S_{\text{em}}(J^l)|\gamma_0\rangle$$

In our case this results in the final nonclassical system state, which is the entangled product of e', ν' states and disjoint em field states

$$|f_w\rangle = \sum_{l=0} c_l|f\rangle_l = |f\rangle_\alpha + |f\rangle_0 = \sum_{l=1} M_w(e'_l, \nu'_l)|e'_l\rangle|\nu'_l\rangle|\gamma^f\rangle_l + \mathrm{M}_0|\mathrm{e}\rangle|\nu\rangle|0\rangle \tag{6}$$

where $|\gamma^f\rangle_l = \hat{S}_{em}(J^l)|0\rangle$ The sum over l means the integral over the correlated final e', ν' momenta p'_l, $p'_{l\nu}$. M_0 is the zero-angle amplitude of particles not scattering. All the partial phases ϕ_l are infinite; moreover, as follows from (3), their differences δ_{lm} are divergent as must be the case for the disjoint states:

$$\delta_{lm} = \int \frac{d\tilde{k}\ [J^{l*}_\mu(\mathbf{k})J^l_\mu(\mathbf{k}) - J^{m*}_\mu(\mathbf{k})J^m_\mu(\mathbf{k})]}{2(2\pi)^3} = F(\varphi_{lm}) \int \frac{dk_0}{k_0}$$

where φ_{lm} is the angle between $\mathbf{p}'_l$, $\mathbf{p}'_m$. By this reason in the limit $t = \infty$ this process is formally completely irreversible, because the T-reflection of the sum of such states with indefinite relative phases produces a new state completely different from the initial one.

Then, as already mentioned, for such a disjoint state any measurement of an arbitrary Hermitian $\hat{B}$ will give $\langle f_0|\hat{B}|f\rangle_\alpha = 0$. This means that we have obtained in a QED-based model a final disjoint state with a finite total energy. Its components $|f\rangle_0$ and $|f\rangle_\alpha$ correspond to the different values of the electron polarization λ_e which we intended to measure. As a result these states have all the observable properties of the mixed state which have to appear after this measurement. Note that this result has been obtained for the complete final state without averaging over some subsystem, or regarding it as an unmeasurable environment (Zurek, 1982). Formally this is the main result of our paper; however, it is important to discuss also practical aspects of continuous photon spectral measurements and possible developments of QFT models for real solid-state detectors.

In practice $\hat{B}$ can only be a bounded operator in H_F, because only this case corresponds to photon measurements by finite detector ensemble (Itzykson and Zuber, 1980). Note that practical direct IT observation is impossible even between a single photon $|\mathbf{k}\rangle$ and the vacuum states, as follows from photocounting theory (Glauber, 1963). It has been shown that all em field operators $\hat{B}_\gamma$ which can be measured are functions of $\hat{n}(\lambda, \mathbf{k}) = a(\lambda, \mathbf{k})\, a^+(\lambda, \mathbf{k})$, the photon number operators. But for such operators we have $\langle \mathbf{k}|\hat{B}_\gamma|0\rangle = 0$, and this also holds true for any state with unsharp photon number. To reveal the presence of IT for a single photon state the only possibility is to perform a special premeasurement procedure (PP), namely, $|\mathbf{k}\rangle$ must be reabsorbed by its source Q_γ and the interference of the source states studied for some new observable of the form $\hat{B}_s = a(\mathbf{k})\hat{B}$. Yet, to our knowledge there is no general proof that such PP always exists for multiphoton states with continuous spectra. The famous recurrence theorem is true only for discrete spectra (Bocchieri and Loinger, 1957).

Such PP certainly does not exist for $|f_w\rangle$ states at $t = \infty$, due to the discussed loss of the relative phases between its parts $|f\rangle_l$. Clearly, if the phase differences δ_{lm} are infinite for the sum of the em field states, then their reabsorption will mean that this loss of coherence is transferred to the Q_γ state, which thereafter will become disjoint. But we will give qualitative arguments that such PP probably do not exist also for the states taken at a finite time.

As an example, we will consider a PP layout in which the scattered e, ν are reflected by some very distant mirrors back to the interaction region where they can rescatter again. Then we calculate the electron radiation appearing after three consequent collisions, taking into account also the "internal" electron radiation between the collisions. The Low theorem demonstrates that the em radiation field in the infrared limit for any process is defined solely by the current calculated between the asymptotic in-and out-momentum eigenstates, thereby neglecting intermediate states (Low, 1958). This means that we can apply the calculations of the method described above and in particular the resulting formula (6).

Then the initial em field state restoration is defined by the $\langle 0|\hat{S}_{em}(J^l)|0\rangle$ amplitude of the $|0\rangle$ restoration in the e, ν rescattering, which is nonzero only for $J_\mu(k) = 0$ as follows from (2). Hence, the electron in- and outmomenta must coincide, and from energy conservation the same must be true for ν. So, we have to calculate the probability P for a second-order weak process $i \to v'_l \to v^r_l \to i$. Here v'_l are all possible intermediate states and $v^r{}_l$ are the reflections of v'_l in nondispersive mirrors. The reflection amplitude is supposed to be $M_r = \exp(i\phi_c)$ and can be suppressed. The calculation is simplified by the spherical symmetry of weak scattering, and we obtain, omitting some unessential details,

$$P = \frac{\displaystyle\int |M_w(e'_l, \nu'_l)M_w(e^r_l, \nu^r_l \to e, \nu)|^2 do_\nu}{\displaystyle\iint |M_w(e'_l, \nu'_l)M_w(e^r_l, \nu^r_l \to e_f, \nu_f)|^2 do_\nu do_f}$$
$$= \frac{|\bar{M}_w|^4 o_\nu \delta^3(p_f - p_e)}{|\bar{M}'_w|^4 o_\nu o_f} = 0$$

Here o_ν and o_f denote the phase spaces of the intermediate and final e, ν states, which are reduced to the corresponding electron phase spaces. Hence, o_ν and o_f are isomorphic to the sphere with the radius $r = |\mathbf{p}_e|$ on which the density of the final states is nearly constant. $\bar{M}_w$ and $\bar{M}'_w$ are the expectation values of M_w over the indicated phase spaces, which are assumed to be of the same order. The restoration of the initial state corresponds to a single point $\mathbf{r}_0 = \mathbf{p}_e$ on this surface. Each infinitely close point to $\mathbf{r}_0$ corresponds to another Hilbert space with an infinite number of soft photons. So the zero probability of the initial state restoration can be simply interpreted geometrically: $\mathbf{r}_0$ is a single point in the phase space which has measure zero. Although these arguments are qualitative in character, they demonstrate that the irreversibility of the disjoint state evolution is connected with the principal uncertainty of scattering angles in QM.

It is important to note that such an effect may exist also for the rescattering of the photon states with finite NDF and continuous spectra which belong to H_F. When the proof of the latter is completed, on which we are presently working, the conditions of the observation of collapse in QED can become tight or and not demand the use of UN representations and disjoint states.

In QFT the situations when the particular dynamics makes some operators unobservable are well known. The most famous example is QCD color confinement, where colored charge is the analog of electric charge and the QCD Hamiltonian contains an infrared singularity induced by the massless bosonic gluon (Itzykson and Zuber, 1980). Any attempt to measure colored operators, for example, the quark or gluon momentum, results in a soft gluon production. In a very short time this colored quantum fuses into some number of colorless hadrons and consequently only the hadron operators are the real observables of this theory.

In practice the measurement is performed on localized states (wave packets) and lasts only a finite time. It has been shown that any localized charged state includes an unlimited number of soft photons (Buchholz *et al.*, 1991). This supposes that the structure of localized and nonlocalized states asymptotically coincides and their evolution will result in analogous disjoint final states.

Real detectors are localized solid objects and the considered model cannot be applied directly. Nevertheless, QFT methods have been used very successfully in solid-state physics, and so we can hope that they will permit us to describe the collapse in real detectors. An example of such an approach gives a simple model of the collapse induced by the ferromagnetic phase transition (Mayburov, 1995).

It is well known that solid-state collective excitations interpreted as massless quasiparticles have excitation spectra without a gap, i.e., infrared divergences (Umezawa *et al.*, 1982). These quanta interact with the *em* field, hence, any excitation of a crystal in the vacuum is to relaxed by soft radiation. The main mechanism may be the cascade phonon decay $P \rightarrow P' + \gamma$. So the excitation of a crystal by a measured energetic particle can result in a new disjoint state of the crystal plus an external electromagnetic field. This idea may also be applicable for a finite systems if its surface is regular and may be regarded as a topological defect with an infinite NDF which results in a special kind of boson condensation in the crystal volume (Umezawa *et al.*, 1978).

In conclusion, we have shown that the final states of e–ν scattering in the standard S-matrix limit reveal asymptotically the properties of the mixed state, i.e., the collapse has been performed. This seems not surprising since the classical features of electron bremsstrahlung states have often been stressed (Kibble, 1968a,b). In addition this model can formally describe the radiation decoherence process of the special kind when the system being measured generates its environment from an initial vacuum.

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